RELATIVISTIC ACCRETION

R. D. Blandford
Theoretical Astrophysics, Caltech, Pasadena, CA 91125, USA

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Abstract

A brief summary of the properties of astrophysical black holes is presented. Various modes of accretion are distinguished, corresponding to accretion at rates from well below to well above the Eddington rate. The importance of mass loss is emphasized when the accreting gas cannot radiate and it is asserted that a strong wind is likely to be necessary to carry off mass, angular momentum and energy from the accreting gas. The possible importance of the black hole spin in the formation of jets and in dictating the relative importance of non-thermal emission over thermal radiation is discussed.

1 Introduction

The general relativistic theory of black holes was mostly developed in the decade spanning the discovery of the Kerr (1963) metric to the collation of their properties in the classic text of Misner, Thorne & Wheeler (1973), (which is still a necessary and nearly sufficient reference for most astrophysical purposes). Despite 25 years of perspective, it still seems almost miraculous that equations as complicated as the field equations of general relativity should produce such an elegant solution and that this should have such magical properties. After the relativists did their job so well and assured us that black holes **could** exist, the astrophysicists were left with the rather messier business of demonstrating that black holes **should** exist and of calculating (or guessing) their observable characteristics. At the stellar level, the first task is a problem in stellar evolution. Chandrasekhar (1931, in Cambridge) first showed that there was a maximum mass for a white dwarf; Oppenheimer & Volkoff (1939) did the same, in principle, for neutrons stars although we still do not know its precise value. However, assuming that it is comfortably less than ~ 3 M$_{\odot}$, we know of about eight, securely-measured, compact object masses in binary systems with masses well in excess of the Chandrasekhar and Oppenheimer-Volkov bounds. These are black holes, the argument goes; what else can they be? This is about as strong a degree of "proof" as one typically gets in astronomy and, if one accepts it, the observers have, in turn, done their job. The **existence** of black holes is no longer an issue.

Turning to massive holes, (with masses of millions to billions M$_{\odot}$), it has long been suspected that these lurk in the nuclei of most normal galaxies, (at least those with luminosities $L \sim L^{*}$), and that they become active, (and classified as quasars, Seyferts and radio galaxies and so on), when fueled at an appropriate rate. Noting again that the most secure evidence comes from dynamics, "massive, dark objects", with masses in the range $\sim 2 \times 10^{6}$ M$_{\odot}$ to $\sim 3 \times 10^{9}$ M$_{\odot}$, have been located in the nuclei of over 20 nearby galaxies. Among the astrophysical alternatives, that have been discussed in the past, are "superstars", which are unstable and would be far too luminous, and clusters of compact objects which are quite contrived from an evolutionary point of view and have very short dynamical lifetimes in the best studied examples. Again, the observers have come through and we can conclude, beyond all reasonable doubt, that most large galaxies contain massive holes in their nuclei.

The most pressing current questions now centre around understanding how accreting black holes actually behave *in situ*. (Evolutionary issues are also quite important and provide some constraints on this behaviour.) This type of research is very difficult to approach deductively from pure physics, because it involves so many non-elementary process - magnetohydrodynamical turbulence, radiative transfer, astrophysical particle acceleration and so on. Therefore, it is prudent to adopt a more phenomenological approach and to try to formulate astrophysical models involving techniques that range from order of magnitude estimates to three dynamical numerical hydrodynamical simulations, that can meet the burgeoning observational database in some middle ground.

In what follows, I will first provide a very quick summary of some salient black hole properties and then go on to summarize some properties of Newtonain accretion disks from a slightly idiosyncratic perspective and emphasizing recent progress that has been made in understanding the nature of the internal torque. Next, I will consider some recent ideas on slow accretion onto black holes. These ideas are also relevant to the corresponding problem of fast accretion and here, some possibilities are sketched. Another topic, of contemporary interest, which I briefly discuss, is the role of black hole spin energy in non-thermal emission from, and jet formation by, black holes.

2 Summary of Black Hole Properties

Astrophysical black holes, (at least those currently observed), form a two parameter family. They are characterized firstly by their gravitational mass M (as measured by the orbital period of a distant satellite), which provides a scale of length and time through the combinations $GM/c^2 \equiv m$ and GM/c^3, respectively. Numerically,

$$m \equiv 1.5 \left(\frac{M}{M_\odot}\right) \mathrm{km} \equiv 5 \left(\frac{M}{M_\odot}\right) \mu\mathrm{s}. \tag{1}$$

It also furnishes a natural scale for luminosity, the Eddington luminosity $L_{\mathrm{Edd}} = 4\pi GMc/\kappa_T$, where κ_T is the Thomson opacity and an associated characteristic accretion rate, $\dot{M}_{\mathrm{Edd}} \equiv L_{\mathrm{Edd}}/c^2$. (Note that we are not concerned with any quantum mechanical features of black holes like Hawking radiation or string entropy. Astrophysical black holes are far too large for these effects to be relevant. They are also far too large for any electrical charge that they may carry to be of gravitational significance.)

The second parameter is of both geometrical and dynamical significance. Traditionally, it is chosen to be the spin angular momentum per unit mass of the hole (expressed as a length in units of c) and denoted by a. This is the quantity that appears in the Boyer-Lindquist form of the metric (*e.g.* Misner, Thorne & Wheeler 1973) and is bounded above by m. Operationally, it can be measured by the precession of a distant gyroscope. However, it is often convenient to use, instead, the angular velocity of the hole, (also measured in units of c as reciprocal length) and which I shall call Ω. The relation between these two quantities is given by

$$\Omega = \frac{1 - (1 - a^2/m^2)^{1/2}}{2a} < \frac{1}{2m} \tag{2}$$

Ω is the angular frequency that an observer at infinity would ascribe to experimentalists hovering just outside the event horizon (provided that she could overcome the strong redshift and see him) (*eg* Shapiro & Teukolsky 19). These two parameters fully characterize the geometry of a black hole where the gravitational effect of the surrounding matter is ignorable and spacetime is asymptotically flat.

Given the Kerr metric, we can compute the orbits of material particles (and photons). The simplest case is circular orbits in the equatorial plane and this is relevant to the structure of thin accretion disks. These have period (measured by a distant observer)

$$P_K = 2\pi(r^{3/2}/m^{1/2} + a) \tag{3}$$

when prograde. These orbits are stable for radii $r > r_{\rm ms}$, the marginally stable circular orbit. The corresponding binding energy of this orbit is denoted $e_{\rm ms}$ and increases from 0.06 to 0.42 c^2 per unit mass as a increases from 0 to m. It is commonly supposed that gas spirals inward through the discuss towards the horizon under the action of viscous stress releasing this binding energy locally as radiation until $r = r_{\rm ms}$, when it plunges quickly into the hole. For this reason, accreting gas is widely thought to release energy at a rate $\sim 10^{20}$ erg g^{-1}.

Non-equatorial orbits are more complex. The most important effect is that their orbital angular momentum will precess about that of the hole with a Lense-Thirring precession frequency, given, to lowest order, by

$$\Omega_{\rm LT} = \Omega(r/2m)^{-3} \tag{4}$$

One of the more remarkable features of black holes is that they are not truly black. A sizeable fraction of their mass energy can be associated with their rotation and is extractable both in principle and, I assert, in practice. To make this plausible, (though not actually *prove* anything), allow our experimentalists hovering just outside the horizon to be sufficiently thoughtful and to change the spin angular momentum of the black hole S, measured in units of G/c^3, by sending tiny packets of mass energy across the horizon. This may be in the form of particles, photons, electromagnetic field etc. Now, if the angular momentum is introduced with angular velocity ω, we have

$$dm = \omega dS = \omega d(am) \tag{5}$$

It seems reasonable that if the observers add this angular momentum with angular velocity $\omega = \Omega$, then there will be no dissipation. (Just consider applying a torque to the surface of a spinning disk.) If we spin up a black hole from rest in this fashion, we can substitute Eq. 2 and integrate differential Eq. 5 to obtain

$$\frac{m^2}{2}[1 + (1 - a^2/m^2)^{1/2}] = {\rm const} = m_0^2 \tag{6}$$

where m_0 is the initial (or irreducible) mass. The limiting mass to which we can spin up the hole in this manner is $2^{1/2}m_0$ when $a = m$. Equivalently, a little algebra shows that

$$m_0 = m(1 + 4\Omega^2 m^2)^{-1/2} \geq 2^{-1/2} m \tag{7}$$

This change is reversible. As Penrose (1969) first showed, there exist negative energy particle orbits that cross the horizon and particles on them *decrease* the mass of the hole. It is then possible to reduce the spin to zero and then return the mass to m_0. All of this becomes more interesting if we use the Kerr metric to compute the area of the horizon and find that it equates to $A = 16\pi m_0^2$. Therefore, reversible processes are those that leave the area of the horizon unchanged. If we change the angular momentum with $\omega \neq \Omega$, the area increases, consistent with its interpretation as being proportional to the entropy of the hole. (Amusingly, if we define an effective radius $r_0 = (A/4\pi)^{1/2} = 2m_0$, and define a rotational speed $\beta = \Omega r_0/c$, we can derive the quasi-Newtonian relation $a = r_0^2\Omega$ and the quasi-special relativistic identity $m = m_0(1 - \beta^2)^{-1/2}$.)

We can therefore extract an energy, up to the difference between the gravitational and the irreducible masses, $m - m_0$, which can be as large as $0.29m$, from a spinning hole, through Penrose-style processes. However, it turns out that this extraction of energy is unlikely to be realized using particles, because it is hard to confine them to the requisite negative energy orbits. The situation is far more promising with ordered magnetic field that is supported by external current (*eg* Thorne, Price & MacDonald 1986 and references therein). The magnetic field lines can thread the horizon of a spinning black hole. A very strong electromotive force will be induced which will make the vacuum into an essentially perfect conductor, (*eg* through pair-production by γ-rays), so that the field lines become equipotentials. Currents will flow and angular momentum and energy will be exchanged with the hole. The relevant angular velocity, ω, is that with which our experimentalist must move so that the electric field vanishes. (If the experimentalist insists upon maintaining a constant distance from the hole, then this can only be accomplished within a finite range of radial coordinate.) We can think of this as the angular velocity of the magnetic field lines and, in a steady state, it must be constant along a given field line. The actual value of this angular velocity depends upon the boundary conditions. Under typical circumstances it is roughly $\omega \sim 0.5\Omega$. In the frame of an experimentalist hovering above the horizon, with an angular velocity

$\omega < \Omega$, a Poynting flux of electromagnetic energy will be seen to enter the hole. However, when we transform into the frame non-rotating with respect to infinity, we must also include the rate of doing work by the electromagnetic torque and we are left with an outwardly directed energy flux that is conserved along a flux tube. Roughly half of the spin energy of a hole may be extracted in this manner; the remainder ending up within the horizon as an increase in the irreducible mass.

3 Newtonian Accretion Disks

First, we review some principles that can be abstracted from the discussion in, for example, Frank, King & Raine (1992), Shapiro & Teukolsky (1983), Pringle (1981), Kato, Fukue & Mineshige (1998) and Holt & Kallman (1998). Consider a thin disk accretion with angular velocity Ω, inflow speed $v \ll \Omega r$, disk mass per unit radius μ and specific angular momentum ℓ. (Henceforth, we measure all radii in units of m.) In assuming that the disk is thin, we are implicitly supposing that the gas can remain cold by radiating away its internal energy. Let the torque that the disk interior to radius r exerts upon the exterior disk be $G(r)$. The equations of mass and angular momentum conservation are then

$$\frac{\partial \mu}{\partial t} = \frac{\partial \mu v}{\partial r}; \qquad \frac{\partial \mu \ell}{\partial t} = \frac{\partial \mu v \ell}{\partial r} - \frac{\partial G}{\partial r}. \tag{8}$$

These equations immediately imply

$$\frac{\partial G}{\partial r} = \frac{\mu v \ell}{2r}; \qquad \frac{\partial \mu}{\partial t} = 2\frac{\partial}{\partial r} r^{1/2} \frac{\partial G}{\partial r} \tag{9}$$

where we have assumed the Keplerian relation $\ell = r^{1/2}$ (*cf* Lynden-Bell & Pringle 1974).

We can combine Eq. 9 to obtain an energy equation

$$\frac{\partial \mu e}{\partial t} + \frac{\partial(\Omega G - \mu v e)}{\partial r} = G\frac{\partial \Omega}{\partial r} \tag{10}$$

where $e = -\Omega\ell/2$ is the Keplerian binding energy, the sum of the kinetic and potential energy per unit mass. (Note the presence of a contribution to the energy flux from the rate at which the torque G, does work on the exterior disk.) The right-hand side represents a radiative loss of energy. Evaluating it, we find that the local radiative flux, in a stationary disk, is three times the rate of local loss of binding energy (Lynden-Bell, Thorne in Pringle & Rees 1972).

Next consider the opposite limiting case when the gas cannot cool and there is no extraneous source or sink of energy. Adding thermodynamic terms to the energy equation, we obtain

$$\frac{\partial\mu(e+u)}{\partial t}+\frac{\partial(\Omega G-\mu v(e+h))}{\partial r}=G\frac{\partial\Omega}{\partial r}+\mu T\frac{ds}{dt} \tag{11}$$

where u is the vertically-averaged internal energy density, h is the enthalpy density, and s is the entropy density (*e.g.* Landau & Lifshitz 1987). As there are no sources or sinks of energy, the right-hand side must vanish:

$$\mu T\frac{ds}{dt}=T\left[\frac{\partial\mu s}{\partial t}-\frac{\partial\mu v s}{\partial r}\right]=-G\frac{\partial\Omega}{\partial r}. \tag{12}$$

As the gas has pressure, we must also satisfy the radial equation of motion:

$$\frac{\partial v}{\partial t}-v\frac{\partial v}{\partial r}+\Omega^2 r=\frac{1}{r^2}+\frac{1}{\rho}\frac{\partial P}{\partial r}. \tag{13}$$

3.1 Magnetic torques

In order to make further progress, it is necessary to specify the torque, G. A traditional prescription, (Shakura & Sunyaev 1973), is to suppose that the shear stress acting in the fluid is directly proportional to the pressure, with constant of proportionality α. In this case,

$$G=2\pi\alpha r^2\int dzP. \tag{14}$$

Traditionally, it has been supposed that $\alpha\sim 0.01-0.1$ on the basis of unconvincing theoretical and observational arguments. However, in recent years a hydromagnetic instability has been rediscovered (Balbus & Hawley 1998, and references therein) and it is clear that ionized disks will generate a dynamically important internal magnetic field on an orbital timescale. The nature of the linear instability can be understood by considering a weak, vertical magnetic field line threading the disk. If gas in the midplane is displaced radially outward, it will drag the magnetic field along with it. (It is a consequence of electromagnetic induction in the presence of an excellent conductor, like an ionized accretion disk, that magnetic field appears to be frozen into the fluid.) As angular momentum is conserved, the displaced fluid element will lag and stretch the magnetic field lines. The magnetic tension associated with the magnetic field will have an azimuthal component which will further increase the angular momentum of the displaced gas and push it further out, amplifying the instability. (A similar effect is exhibited by a tethered, artificial satellite.)

The non-linear development of this instability has been investigated numerically and although many uncertainties remain, it appears that the traditional prescription for α is not wildly wrong at least as long as the disk is ionized. (Empirically, it appears that predominantly neutral disks, as our found in young stellar objects, for example, exhibit lower values of α.) A major unsolved problem is the nature of the torque when the accretion rate is large enough for the radiation to be trapped by Thomson scattering, so that the disk fluid becomes radiation-dominated, like the early universe. Under these conditions, we expect the short wavelength modes to be damped by radiation drag and radiative viscosity and the longer wavelength components may escape through buoyancy (Agol & Krolik 1998 and references therein). More numerical simulations, including radiation transfer, are necessary to help us understand what actually happens.

As we have just emphasized on general grounds, an internal torque in a shearing medium inevitably leads to dissipation. In the case of MHD torques in an accretion disk, it has been argued that this happens through a hydromagnetic turbulence spectrum which ends up with the ions being heated by a magnetic variant on Landau damping called transit time damping (Quataert & Gruzinov 1998). This is not the only possibility. It is conceivable that magnetic reconnection or non-local dissipation in an active, accretion disk corona may also play a role.

4 Slow Accretion

4.1 ADAF solution

There has been much attention in recent years to the problem of slow accretion. Observationally, this is motivated by the discovery that many local galactic nuclei are conspicuously under-luminous. A good example is our Galactic center, where the mass supply rate may be as high as $\sim 10^{22}$ g s^{-1} and the bolometric luminosity may be as low as $\sim 10^{36}$ erg s^{-1}, giving a net efficiency of $\sim 10^{14}$ erg g^{-1}, $\sim 10^{-7}c^2$, (and quite unlikely to exceed $\sim 10^{-4}c^2$), a far cry from the naive expectation of standard disk accretion. As discussed by Narayan & Yi (1994) and Kato *et al* (1998), and many references therein, one possible resolution of this paradox is that the gas flows in to the hole as an "Advection-Dominated Accretion Flow", or ADAF for short. In order for this flow to be established, it is necessary that the gas not be able to cool on the inflow timescale. This, in turn, requires that the viscosity be relatively high and that the hot ions, which can achieve temperatures as high as ~ 100 MeV, only heat the electrons by Coulomb interaction. (Ultrarelativistic electrons are very efficient radiators.)

The basic idea and assumptions are set out most transparently in Narayan & Yi (1994; cf. also Ichimaru 1977, Abramowicz et al. 1995). In the simplest, limiting case, it is assumed that there is a stationary, one-dimensional, self-similar flow of gas with $\mu \propto r^{1/2}$, $\Omega \propto r^{-3/2}$, and $v, a \propto r^{-1/2}$, where $a = [(\gamma - 1)h/\gamma]^{1/2}$ is the isothermal sound speed and the radial velocity $v << \Omega r$. The requirement that $P \propto r^{-5/2}$ transforms the radial equation of motion into

$$\Omega^2 r^2 - \frac{1}{r} + \frac{5a^2}{2} = 0. \tag{15}$$

Conservation of mass, momentum and energy gives

$$\mu v \equiv \dot{M} = \text{constant} \tag{16}$$

$$\dot{M} r^2 \Omega - G = F_\ell \tag{17}$$

$$G\Omega - \dot{M} Be = F_E \tag{18}$$

where the inwardly directed angular momentum flux, F_ℓ, and the outwardly directed energy flux, F_E, are constant if there are no sources and sinks of angular momentum or energy. ($Be = \Omega^2 r^2/2 - 1/r + h$ is the Bernoulli constant.) Now, the terms on the left-hand side of Eq. 17 scale $\propto r^{1/2}$ and those of Eq. 18 scale $\propto r^{-1}$. Therefore, if we require the flow to be self-similar over several decades of radius, both constants must nearly vanish. In the limit, $F_\ell = F_E = 0$.

Combining equations, we solve for the sound speed a and the Bernoulli constant.

$$a^2 = \left[\frac{3(\gamma - 1)}{5 - 3\gamma}\right] \Omega^2 r^2 = \frac{6(\gamma - 1)}{(9\gamma - 5)r} \tag{19}$$

$$Be = \Omega^2 r^2. \tag{20}$$

The elementary ADAF solution is then completed by defining an α viscosity parameter through, $G = \dot{M} r^2 \Omega = \alpha \mu r a^2$, which then implies $v = \alpha a^2/\Omega r$, assuming that $\alpha \ll (5/3 - \gamma)^{1/2}$.

There are concerns with this solution, as identified by Narayan & Yi (1995). The most important of these is the worry that the accreting gas may not be bound to the black hole. This can be demonstrated by observing that the Bernoulli constant, Be is generically positive due to the viscous transport of energy. This means that an element of gas has enough internal energy, (taking into account the capacity to perform PdV work), to escape freely to infinity). In the particular case when the specific heat ratio is $\gamma \sim 5/3$, as it will be if only the ions are effectively heated, note that the the self-similar solution is nearly non-rotating. A lot of angular momentum and orbital kinetic energy must be lost at some outer radius, where the ADAF solution first becomes valid. (This is called the transition radius.) As the gas cannot cool here, by assumption, there seems nowhere for the energy to go except in driving gas away. Another precarious part of the ADAF solution is found close to the rotation axis. It is proposed that when the viscous torque is relatively large, that the flow extend all the way to the polar axis (Narayan & Yi 1995). This removes one exposed surface, but it does so at the expense of creating a stationary column of gas, which cannot be supported at its base. It is unlikely to persist.

4.2 ADIOS solution

For these reasons, Blandford & Begelman (1998) have proposed a variant upon the ADAF solution called an "Advection-Dominated Inflow Outflow Solution". Here the key notion is that the excess energy and angular momentum is removed by a wind at all radii. Again it is simplest to assume self-similarity. We follow the Narayan & Yi (1984) solution, but supplement it by allowing the mass accretion rate to vary with radius.

$$\dot{M} \propto r^p; \qquad 0 \leq p < 1. \tag{21}$$

The mass that is lost from the inflow escapes as a wind. If we adopt self-similar scalings, and use the above definitions of the flow of angular momentum and energy, we can write,

$$F_\ell = (\dot{M} r^2 \Omega - G) = \lambda \dot{M} r^{1/2}; \qquad \lambda > 0. \tag{22}$$

and

$$F_E = G\Omega - \dot{M}\left(\frac{1}{2}\Omega^2 r^2 - \frac{1}{r} + \frac{5a^2}{2}\right) = \frac{\epsilon \dot{M}}{r}; \qquad \epsilon > 0. \tag{23}$$

where λ, ϵ, like p are constants that can be fixed Equivalently, for the specific angular momentum and energy carried off by the wind, we have

$$\frac{dF_\ell}{d\dot{M}} = \frac{\lambda(p+1/2)r^{1/2}}{p}; \qquad \frac{dF_E}{d\dot{M}} = \frac{\epsilon(p-1)}{pr} \tag{24}$$

With these modifications, the radial equation of motion becomes

$$\Omega^2 r^2 - \frac{1}{r} + (5/2 - p)a^2 = 0. \tag{25}$$

Similarly, the Bernoulli constant becomes

$$Be = \frac{\Omega^2 r^2}{2} - \frac{1}{r} + \frac{5a^2}{2} = pa^2 - \frac{1}{2}\Omega^2 r^2 \tag{26}$$

and it can now have either sign. (A limit must be taken to recover Eq. 20.) Combining these equations, we obtain

$$\Omega r^{3/2} = \frac{(5-2p)\lambda}{15-2p} + \frac{[(5-2p)^2\lambda^2 + (15-2p)(10\epsilon + 4p - 4\epsilon p)]^{1/2}}{15-2p} \tag{27}$$

It is a matter of algebra to complete the solution and determine how the character of the solutions depends upon our three independent, adjustable parameters, p, λ, ϵ.

Let us consider some special cases.

1. $p = \lambda = \epsilon = 0$. There is no wind and the system reduces to the non-rotating Bondi solution.

2. $p = \lambda = 0$, $\epsilon = 3(1-f)/2$. This corresponds to flow with no wind but with radiative loss, which carries away energy but not angular momentum. The parameter f, (Narayan &Yi 1994), is defined by the relation $\dot{M}TdS/dr = fGd\Omega/dr$.

3. $p = 0$, $\lambda = 1$, $\epsilon = 1/2$. This describes a magnetically-dominated wind with mass flow conserved in the disk. All of the angular momentum and energy is carried off by a wind with $dF_E/dF_\ell = \Omega$ (cf. Blandford & Payne 1982, Königl 1991). There is no dissipation in the disk, which is cold and thin.

4. $\lambda = 2p[(10\epsilon + 4p - 4\epsilon p)/(2p+1)(4p^2 + 8p + 15)]^{1/2}$. This corresponds to a gas dynamical wind where $dF_\ell/d\dot{M} \equiv \ell_W = r^2\Omega \equiv \ell$. The wind carries off its own angular momentum at the point of launching and does not exert any reaction torque on the remaining gas in the disk. Any magnetic coupling to the disk implies $\ell_W > \ell$.

5. $ra^2 = r^3\Omega^2/2p = 1/(p+5/2)$. This corresponds to a marginally bound flow with vanishing Bernoulli constant. In practice, it is expected that $Be < 0$. In the limiting case, a single proton at the event horizon can, altruistically, sacrifice itself to allow up to a thousand of its fellow protons to escape to freedom from $\sim 1000m$.

What this exercise demonstrates is that gas can accrete slowly onto a black hole without radiating, provided it uses loses enough mass, energy and angular momentum and that the rate of mass accretion by the black hole may be very much less than the mass supply rate. (This has implications for the rate of balck hole growth due to accretion in the early universe.) In order to go beyond this, we must introduce some additional physics into our discussion of the disk and the wind.

5 Fast Accretion

The solution, that I have just described, is appropriate when the accretion rate is slow enough that the gas cannot cool radiatively. How slow this must be depends upon microphysical details that are still uncertain. However, it appears that the underlying physical principles are still appropriate in the opposite limiting case of fast accretion. In this limit, as the accretion rate is high the density is sufficiently large to allow the gas to come into local thermal equilibrium and to emit radiation so that the photons dominate the gas pressure. However, the density is also large enough for the gas to become optically thick to Thomson scattering and for the radiation to be trapped. Under such circumstances, photons will random walk relative to the gas with a characteristic speed $\sim c/\tau_T$, where τ_T is the optical depth to Thomson scattering. If the density is so large that this speed is less than the bulk speed of the gas, then the photons are essentially trapped and, once again, the gas is prevented from radiative cooling. Typically, this occurs if $\dot{M} > \dot{M}_{\rm Edd}$.

It is therefore possible to develop a model of accretion onto a black hole in the limit when $\dot{M} >> \dot{M}_{\rm Edd}$ (Begelman & Blandford in preparation).

We treat disk accretion in much the same way as we treat it in the ion-dominated case, with the unimportant difference that the effective specific heat ratio (not the true one) is $\gamma = 4/3$. In order to define a vertical structure for the disk, we have to make some assumption about the angular velocity distribution. One possibility is that the angular velocity is constant on cylinders. This is equivalent to assuming that the equation of state is barytropic. A much better assumption, and one that has some physical support, is that the disk is marginally unstable to convective overturn (*eg* Begelman & Meier 1982). This implies that the entropy is constant on surfaces of constant angular momentum - the "gyrentropic hypothesis" (*cf* Abramowicz & Paczyński 1982, Blandford, Jaroszyński & Kumar 1985). This allows entropy and angular momentum to be freely transported along these surfaces; transport perpendicular to these surfaces requires additional (and presumably magnetic) viscous stress.

The attached wind is also radiation-dominated and it is possible to find self-similar solutions that describe a wind that carries off the mass, angular momentum and energy released from the surface of the disk. Eventually, this outflow will become tenuous enough to allow the trapped radiation to escape. There may be a third region, where the flow is optically thin, and where the gas may start to recombine so that it can be accelerated by line radiation pressure.

Super-critical accretion flows, with $\dot{M} >> \dot{M}_{\rm Edd}$, almost surely occurs naturally in both Galactic sources like SS433 and GRS 1915+112 and in extragalactic sources like the radio-quiet quasars and the broad absorption line quasars. In both circumstances, it appears that the rate of mass outflow exceeds the Eddington rate by a large factor. Presumably, the same is true of the rate of mass supply.

6 The Importance of Spin

As emphasized above, there are two potential power sources, the binding energy released by accreting gas and the spin energy of the central black hole. It is natural to associate the former with "thermal" emission and the latter with "non-thermal" emission and this separation has provided the basis for a variety of "unified", (and "grand unified") models of AGN over the past twenty years. It is apparent that the electromagnetic extraction of energy and angular momentum from the black hole can occur in principle. A question of recent interest is "How much does this occur in practice?".

Clearly, there are two requirements beyond the presence of the black hole, spin and magnetic flux, and on these we can only speculate. It is widely assumed that freshly formed holes, and those that have recently undergone major mergers, spin rapidly in the sense that $\Omega m \sim 0.2 - 0.5$. However, subsequent addition of angular momentum (eg through rapid episodes of accretion or minor mergers) may be stochastic, as opposed to the mass which increases secularly. This can lead to a spin down. Alternative a strong dynamical interaction without a surrounding warped disk (as proposed by Natarajan & Pringle, 1998, preprint) may lead to a very rapid de-spinning of the hole without the creation of much non-thermal energy.) It has also been suggested (Ghosh & Abramowicz 1998, Livio, Ogilvie & Pringle 1998, *cf* Blandford & Znajek 1977) that the total electromagnetic power that derives from the hole is very small compared with that which derives from the disk. The basis of this argument is that the strength of the magnetic field that threads the hole is likely to be no larger than that threading the disk and the area of the disk is much larger than that of the hole. This, indeed, may be the case in the majority of active nuclei (in particular radio quiet-quasars and Seyfert galaxies) which are not non-thermal objects.

However, it is not guaranteed that these limits always apply. For example, the strength of the magnetic field interior to the disk is really only limited by the Reynolds' stress of the orbiting gas just as is supposed to occur at the Alfvén surface surrounding an accreting, magnetized neutron star. Alternatively, the magnetic field associated with the disk may be predominately closed with little radial component (as it is being strongly sheared) so that it does not extract much energy, but can provide pressure. Under either of these circumstances the non-thermal power extracted from the hole can be dominant. What actually occurs depends upon issues of stability and supersonic reconnection.

There is a strong observational incentive to consider these processes. It has become clear that some sources are spectacularly non-thermal. Bulk Lorentz factors in excess of 10 are required to explain some superluminal motion and, perhaps, much larger relativistic speeds are indicated by the intraday variable sources. The γ-ray jets discovered by the EGRET instrument on Compton Gamma Ray Observatory can be prodigiously energetic and in some cases, seem to transport far more energy, even allowing for relativistic beaming, than is observed in the remainder of the electromagnetic spectrum (Hartmann *et al* 1996). The rapidly variable X-rays produced by the Galactic superluminal sources are far too energetic to be the result of black body-emission from the surface of an accretion disk. To this reviewer, at least, it is unlikely that this power can derive solely from an active disk corona. There is ample spin energy associated with the hole to account for the observations and an environment where thermalization will be very difficult. The assocation of the jet power and the high energy emission with the black hole is surely as suggestive as, historically, was the association of the Crab Nebula with PSR 0531+21.

There is another interesting possibility (Blandford & Spruit 1998, in preparation, *cf* also Livio, these proceedings). This is that magnetic field attached to the inner disk may also connect to the black hole. Now if $\Omega > 0.093/m$, the hole rotates faster than the gas in the marginally stable circular orbit and even faster than all the gas beyond this. Therefore, unless the hole is very slowly rotating, a magnetic connection will transport angular momentum radially outward. As the hole has a much higher effective resistivity than the disk, we can regard the field lines as being effectively transported by the disk. Therefore, they will only do mechanical work on the disk with no direct dissipation. If this interaction is strong enough the increase in the angular momentum of the gas can be enough to reverse the accretion flow, driving some gas radially outward while a fraction falls inward. This can happen in a quasi-cyclical manner and it is tempting to associate some of the quasi-periodic behavior observed in Galactic black hole binaries, notably GRS 1915+112 with just this sort of process.

7 Conclusion

I hope that this somewhat cursory description of recent developments is sufficient to persuade the reader that the study of black holes both in our Galaxy, in the nuclei of nearby galaxies and in distant quasars, is on the ascendant. Measurements of mass and (possibly) spin are helping make astrophysical arguments quantitative. Observations of quasi-periodicity, notably by RXTE (Greiner, Morgan & Remillard 1996), are strongly suggestive of non-linear dynamical processes in the curved spacetime close to the black hole. Direct measurement of iron lines and their profiles explore the surfaces of inner accretion disks. Jets, are now observed to be commonplace in accreting systems and, in the case of black hole systems must derive from close to the hole (as close as $\sim 60m$ inthe case of M87, Junor & Biretta 1995).

The best is yet to come. There is a suite of X-ray telescopes planned for launch in the coming years, AXAF, XMM and ASTRO E. There are ambitious plans for superior instruments, like GLAST and CONSTELLATION-X to be launched during the coming decade and for space-based VLBI to be developed in earnest. On an even longer timescale, the space-based gravitational radiation detection, LISA has the projected capability to detect merging black holes from cosmological distances and to provide direct quantitative tests of strong field general relativity for the first time. On the theoretical front, the numerical capability to perform large scale, three dimensional, hydromagnetic simulations is already here and the incorporation of radiative transfer and credible dissipative processes is a not so distant hope. This is a good time for a student to start research in black hole astrophysics.

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